GCE AS/A level

0975/01

S16-0975-01

MATHEMATICS – C3
Pure Mathematics

A.M. WEDNESDAY, 8 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^{\frac{\pi}{5}} e^{\tan^2 x}\,dx.$$

Show your working and give your answer correct to five decimal places. [4]

(b) **Use your answer to part *(a)*** to deduce an approximate value for the integral

$$\int_0^{\frac{\pi}{5}} e^{\sec^2 x}\,dx.$$ [2]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$3\,\mathrm{cosec}\,\theta\,(\mathrm{cosec}\,\theta - 1) = 5\cot^2\theta - 9.$$ [6]

(b) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$2\,\mathrm{cosec}\,\phi + 3\sec\phi = 0.$$ [3]

3. The curve C is defined by

$$x^2 + 3xy + 2y^3 - 2x = 21.$$

The point P has coordinates (– 5, 2) and lies on C.

Find the value of $\frac{dy}{dx}$ at P. [4]

4. A function is defined parametrically by

$$x = 4\sin 3t,\ y = 2\cos 3t.$$

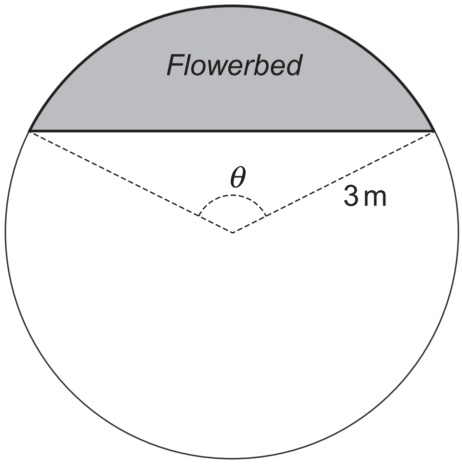
(a) Find and simplify an expression for $\frac{dy}{dx}$ in terms of t. [4]

(b) Find and simplify an expression for $\frac{d^2y}{dx^2}$

(i) in terms of t,

(ii) in terms of y. [4]

5. The diagram shows a circular garden plot of radius 3 m. Alun wants to use a minor segment of the plot as a flowerbed and has a 13·5 m length of edging, all of which he intends to use to form the perimeter of the shaded area below. The angle subtended at the centre of the circular plot is denoted by θ radians.



(a) Show that θ satisfies the equation

$$\theta + 2\sin\left(\frac{\theta}{2}\right) = 4{\cdot}5\,. \qquad [3]$$

(b) Alun believes that the value of θ will turn out to be approximately 2·5.
Starting with θ_0 = 2·5, use the recurrence relation

$$\theta_{n+1} = 4{\cdot}5 - 2\sin\left(\frac{\theta_n}{2}\right)$$

to find the values of θ_1, θ_2, θ_3. Write down the value of θ_3 correct to two decimal places and prove that this is the value of θ correct to two decimal places. [5]

6. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $\ln(\cos x)$ [3]

(b) $\tan^{-1}(\frac{x}{3})$ [3]

(c) $e^{6x}(3x - 2)^4$ [4]

7. *(a)* Find each of the following, simplifying your answer wherever possible.

(i) $\int 7e^{5-\frac{3}{4}x}\,dx$ (ii) $\int \sin\left(\frac{2x}{3} + 5\right)dx$ (iii) $\int \frac{8}{(9-10x)^3}\,dx$ [6]

(b) Given that a > 0 and that

$$\int_a^6 \frac{1}{4x+3}\,dx = 0{\cdot}1986,$$

find the value of the constant a. Give your answer correct to one decimal place. [5]

TURN OVER

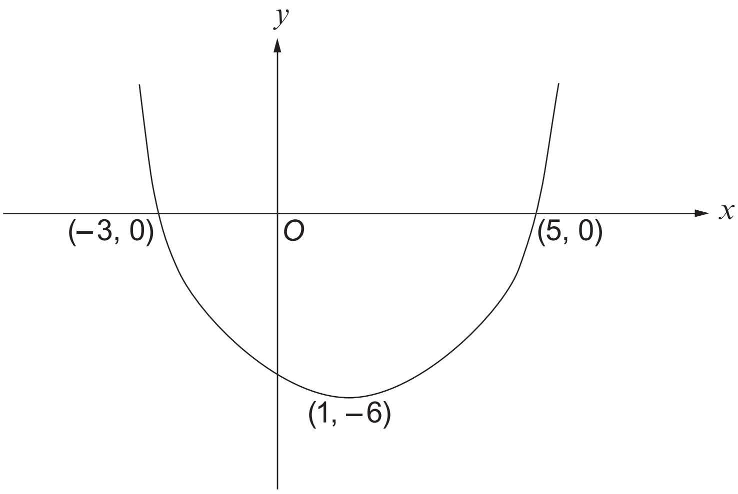
8. *(a)* Show, by counter-example, that the following statement is false.

'If the integers a, b, c, d are such that a is a factor of c and b is a factor of d, then $(a + b)$ is a factor of $(c + d)$.' [2]

(b) Solve the equation

$$|5x + 4| = -7x.$$ [4]

(c) The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-3, 0)$ and $(5, 0)$ and has a minimum point at $(1, -6)$.



(i) The graph of $y = 4f(x + a)$ passes through the origin. Write down the possible values of a.

(ii) The y-coordinate of the stationary point on the graph of $y = bf(x + 2)$ is 4. Write down the value of b. [2]

9. The function f has domain $(-\infty, 12]$ and is defined by

$$f(x) = e^{4 - \frac{x}{3}} + 8.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

10. The function h is defined by

$$h(x) = \frac{4x+3}{5x-4}.$$

(a) Show that $hh(x) = x$. [3]

(b) **Use the result of part *(a)*** to write down an expression for $h^{-1}(x)$.

Hence evaluate $h^{-1}(-1)$. [2]

END OF PAPER